

On Coulomb drag in double layer systems

Bruno Amorim¹ and N M R Peres²

¹ Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, E-28049 Madrid, Spain.

² Physics Department and CFUM, University of Minho, P-4710-057, Braga, Portugal.

E-mail: amorim.bac@icmm.csic.es

Abstract. We argue, for a wide class of systems including graphene, that in the low temperature, high density, large separation and strong screening limits the drag resistivity behaves as d^{-4} , where d is the separation between the two layers. The results are independent of the energy dispersion relation, the dependence on momentum of the transport time, and the electronic wave function structure. We discuss how a correct treatment of the electron-electron interactions in an inhomogeneous dielectric background changes the theoretical analysis of the experimental drag results of Kim *et al* (2011 *Phys. Rev. B* 161401). We find that a quantitative understanding of the available experimental data (Kim *et al* 2011 *Phys. Rev. B* 161401) for drag in graphene is lacking.

PACS numbers: 72.80.Vp

1. Introduction

Coulomb drag [1, 2] occurs when driving a current through a metallic layer, referred to as the active layer (and which will be denoted by 2), induces a current in another metallic layer, separated by a distance d , referred to as the passive layer (and which will be denoted by 1). This phenomenon is caused by the transference of momentum between electrons in different layers due to the interlayer electron-electron interaction. In experimental situations no current is allowed to flow in the passive layer, so that an electrical field, E_1 , builds up in that layer. In this situation, the drag resistivity, ρ_D , is obtained from the ratio

$$\rho_D = \frac{E_1}{j_2}, \quad (1)$$

where j_2 is the current driven through the active layer.

Recently there has been great deal of interest on the phenomenon of Coulomb drag in graphene double layers. Although the number of experimental works on drag in graphene is still scarce [3, 4, 5], there are already plenty of theoretical works on the topic [6, 7, 8, 9, 10, 11, 12]. However, among the theoretical literature we find contradictory statements, particularly in the limit of low temperature, high density, large layer separation and strong screening. In this limit, it is a well established result that the drag resistivity between two 2DEG's (two dimensional electron gases), with parabolic dispersion relations and a constant transport time, depends on temperature as T^2 and on the interlayer separation as d^{-4} [13] (a result previously given in [14] without derivation). Graphene is distinct from the 2DEG in three ways: (i) the low energy dispersion relation is linear, instead of parabolic; (ii) the electron wave function has a spinorial structure, instead of being a scalar; (iii) the transport time for the dominant kind of impurities is proportional to the momentum, $\tau(k) \propto k$, [15]. There is agreement [7, 8, 9, 10, 11, 12] that in the low temperature/high density limit, the T^2 dependence should still hold in graphene. However, there are a few contradictory results on the dependence of the drag resistivity on the layer separation, and how considering a constant or a momentum dependent transport time changes the result. In this paper we will give special attention to the limit of low temperature, high density, large interlayer separation and strong screening. Therefore, we will make a brief overview of the available results in the literature in this limit.

- (i) Tse *et al* [7] assumes a constant transport time, and a dependence of d^{-4} is obtained.
- (ii) Peres *et al* [8] considers a momentum dependent transport time, $\tau(k) \propto k$, obtaining a d^{-6} dependence (as we explain later this result is due to an algebraic error at the end of the asymptotic calculation; correcting this gives a d^{-4} dependence).
- (iii) Katsnelson [9] considers a constant transport time and obtains a d^{-4} dependence.
- (iv) Hwuang *et al* [10] considers both cases of a constant transport time and a transport time proportional to the momentum, $\tau(k) \propto k$. For a constant transport time a d^{-6} dependence is obtained (in contradiction with the result from [7]). For the

case of a momentum dependent transport time a d^{-4} behaviour is obtained. The case of drag between two bilayer graphene layers is also studied. Using a constant transport time a d^{-4} behaviour is obtained in this same limit.

- (v) Narozhny *et al* [11] considers both cases of a constant and a linearly momentum dependent transport time. For the case of a constant transport time it is obtained a d^{-4} dependence. It is argued that in the low temperature, high density limit, this result still holds, regardless of the momentum dependence of the transport time.
- (vi) Carrega *et al* [12], in a recent independent work, studies drag between massless Dirac electrons. It is proved that in the low temperature/high density limit, the dependence on momentum of the transport time is irrelevant and a d^{-4} dependence is obtained for large interlayer separations.

In this paper, we attempt to clarify this situation by presenting a clear proof that in the limit of low temperature, high density, large interlayer separation and strong screening the drag resistivity should always depend on temperature as T^2 and on distance separation as d^{-4} . Our analysis is independent of the energy dispersion relation, electron wave function structure and dependence on momentum of the transport time. The structure of the paper is as follows: in section 2, we present the general theory of Coulomb drag. In section 3, we present a general argument proving that the drag resistivity in the limit of low temperature, high density, large separation between layers and strong screening should always depend on temperature as T^2 and on the interlayer separation as d^{-4} . We also study the case of graphene for small interlayer separation. In section 4, we specialize to graphene and re-analyze the asymptotic result derived in [8]. Finally, in section 5 we look into the experimental data of [3] using a more careful treatment of the bare Coulomb interactions.

2. General formulation of Coulomb drag

Theoretically, it is convenient to compute conductivities instead of resistivities. For isotropic systems the drag resistivity is related to the conductivities by

$$\rho_D = -\frac{\sigma_{12}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \simeq -\frac{\sigma_{12}}{\sigma_{11}\sigma_{22}}, \quad (2)$$

where σ_{11} and σ_{22} are the intralayer conductivities of the passive and active layers and $\sigma_D \equiv \sigma_{12} = \sigma_{21}$ is the drag conductivity; in (2) it was assumed that $\sigma_{11}, \sigma_{22} \gg \sigma_D$.

Considering that tunnelling between layers does not occur and that the intralayer transport is dominated by impurity scattering, the drag conductivity can be computed in second order in the interlayer interaction using either Boltzmann's kinetic equation [10, 8, 13, 16], the memory-function formalism [17] or Kubo's formula [10, 11, 18], and is given by

$$\sigma_D^{ij} = \frac{e_1 e_2}{16\pi k_B T} \int \frac{d^2 q}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\omega |U_{12}(\vec{q}, \omega)|^2}{\sinh^2(\beta\hbar\omega/2)} \Gamma_1^i(\vec{q}, \omega) \Gamma_2^j(\vec{q}, \omega), \quad (3)$$

where e_a is the charge of carriers in layer a , $U_{12}(\vec{q}, \omega)$ is the interlayer electron-electron interaction and $\Gamma_a^i(\vec{q}, \omega)$ is the i -th component of non-linear susceptibility of layer a . In the weak impurity limit, the non-linear susceptibility reads [7, 10] (the layer index is omitted for simplicity)

$$\vec{\Gamma}(\vec{q}, \omega) = -2\pi g \sum_{\lambda, \lambda'} \int \frac{d^2 k}{(2\pi)^2} f_{\vec{k}, \vec{k}+\vec{q}}^{\lambda, \lambda'} \left[n_F(\epsilon_{\vec{k}, \lambda}) - n_F(\epsilon_{\vec{k}+\vec{q}, \lambda'}) \right] \\ \times \left(\vec{v}_{\vec{k}, \lambda} \tau_{\vec{k}, \lambda} - \vec{v}_{\vec{k}+\vec{q}, \lambda'} \tau_{\vec{k}+\vec{q}, \lambda'} \right) \delta \left(\epsilon_{\vec{k}, \lambda} - \epsilon_{\vec{k}+\vec{q}, \lambda'} + \hbar\omega \right), \quad (4)$$

where g is the flavour degeneracy, λ, λ' are band indices, $f_{\vec{k}, \vec{k}'}^{\lambda, \lambda'} = \left| \langle \vec{k}, \lambda | \vec{k}', \lambda' \rangle \right|^2$ is the electron wave function overlap factor (which encodes the structure of the wave function), $n_F(\epsilon)$ is the Fermi-Dirac distribution function, $\vec{v}_{\vec{k}, \lambda}$ is the particle's group velocity, $\tau_{\vec{k}, \lambda}$ is the impurity transport time, and $\epsilon_{\vec{k}, \lambda}$ is the energy dispersion.

For the interlayer interaction one usually uses the RPA dynamically screened Coulomb interaction [16],

$$U_{12}(\vec{q}, \omega) = \frac{V_{12}(\vec{q})}{\epsilon_{RPA}(\vec{q}, \omega)}, \quad (5)$$

where $V_{ab}(\vec{q})$ is the bare Coulomb interaction between electrons in layer a and b , and $\epsilon_{RPA}(\vec{q}, \omega)$ is the RPA dielectric function for the double layer system, which is given by [16, 19]

$$\epsilon_{RPA}(q, \omega) = [1 - V_{11}(\vec{q})\chi_1(\vec{q}, \omega)] [1 - V_{22}(\vec{q})\chi_2(\vec{q}, \omega)] \\ - V_{12}(\vec{q})V_{21}(\vec{q})\chi_1(\vec{q}, \omega)\chi_2(\vec{q}, \omega), \quad (6)$$

and $\chi_a(\vec{q}, \omega)$ is the polarizability of layer a . The bare Coulomb interactions can in general be written as (see Appendix A)

$$V_{ab}(\vec{q}) = \frac{1}{\epsilon_{ab}(q)} \frac{e^2}{2\epsilon_0 q} e^{-qd(1-\delta_{ab})}, \quad (7)$$

where ϵ_0 is the vacuum permittivity and $\epsilon_{ab}(q)$ are effective dielectric functions. If the metallic layers are immersed in a homogeneous dielectric with constant ϵ_r then $\epsilon_{ab}(q) = \epsilon_r$.

3. Low temperature behaviour

We now study the behaviour of the drag conductivity in the limit of low temperature, $\epsilon_{F1(2)} \gg k_B T$, where $\epsilon_{F1(2)}$ is the Fermi energy of layer 1(2). Unless specified otherwise, we will keep the energy dispersion relation, $\epsilon_{\vec{k}, \lambda}$, the transport time, $\tau_{\vec{k}, \lambda}$, and wave function overlap factors, $f_{\vec{k}, \vec{k}+\vec{q}}^{\lambda, \lambda'}$, general. We assume isotropy and that there is only one band at the Fermi level. Central to the analysis is the realization that the energy dispersion relation close to the Fermi energy is always linear in momentum, that is, the dispersion can be approximated by:

$$\epsilon_{\vec{k}, c} - \epsilon_F \simeq \hbar v_F (k - k_F), \quad (8)$$

where v_F is the slope of the band at the Fermi energy, termed Fermi velocity, and the label c refers to the conduction band. For graphene (8) is exact. We also assume that the two metallic layers are placed in vacuum, such that

$$\epsilon_{RPA}(q, \omega) = 1 + \chi_1(\vec{q}, \omega) \chi_2(\vec{q}, \omega) \left(\frac{e^2}{2\epsilon_0 q} \right)^2 2 \sinh(qd) e^{-qd} - \frac{e^2}{2\epsilon_0 q} [\chi_1(\vec{q}, \omega) + \chi_2(\vec{q}, \omega)]. \quad (9)$$

Due to the factor $\sinh^{-2}(\beta\hbar\omega/2)$ in (3), the main contribution to the integral in ω comes from $\hbar\omega \lesssim k_B T$. Since $\epsilon_{F1(2)} \gg k_B T$, we can therefore expand the remaining integration kernel to lowest order in ω and set $T = 0$. Therefore we replace the dynamically screened dielectric function, $\epsilon_{RPA}(\vec{q}, \omega)$, by its static value, $\epsilon_{RPA}(\vec{q}, 0)$, and expand the non-linear susceptibility of each layer, (4), to lowest order in ω . Using the energy conserving δ -function in (4), $\delta(\epsilon_{\vec{k},\lambda} - \epsilon_{\vec{k}+\vec{q},\lambda'} + \hbar\omega)$, we expand to lowest order in ω :

$$n_F(\epsilon_{\vec{k},\lambda}) - n_F(\epsilon_{\vec{k}+\vec{q},\lambda'}) = n_F(\epsilon_{\vec{k},\lambda}) - n_F(\epsilon_{\vec{k},\lambda} + \hbar\omega) \simeq -\hbar\omega \frac{\partial n_F(\epsilon_{\vec{k},\lambda})}{\partial \epsilon} \simeq \hbar\omega \delta(\epsilon_F - \epsilon_{\vec{k},\lambda}). \quad (10)$$

Therefore $\vec{\Gamma}(\vec{q}, \omega)$ has a linear contribution in ω . Since we want $\vec{\Gamma}(\vec{q}, \omega)$ to lowest order in ω we can now set $\omega = 0$ in $\delta(\epsilon_{\vec{k},\lambda} - \epsilon_{\vec{k}+\vec{q},\lambda'} + \hbar\omega)$, obtaining

$$\vec{\Gamma}(\vec{q}, \omega) = -g \frac{\hbar\omega}{2\pi} \int d^2k f_{\vec{k},\vec{k}+\vec{q}}^{c,c} \delta(\epsilon_F - \epsilon_{\vec{k},c}) \delta(\epsilon_F - \epsilon_{\vec{k}+\vec{q},c}) \times (\vec{v}_{\vec{k},c} \tau_{\vec{k},c} - \vec{v}_{\vec{k}+\vec{q},c} \tau_{\vec{k}+\vec{q},c}). \quad (11)$$

Since we have isotropy we can write $\vec{v}_{\vec{k},c} \tau_{\vec{k},c} = \vec{k} g(k)$, where $g(k)$ is a general function that satisfies $k_F g(k_F) = v_F \tau_F$, τ_F being the transport time at the Fermi level. The δ -functions set $|\vec{k}| = |\vec{k} + \vec{q}| = k_F$, and therefore we can take $g(k)$ outside the integral, obtaining to lowest order in ω

$$\vec{\Gamma}(\vec{q}, \omega) = g \frac{\hbar\omega v_F \tau_F}{2\pi k_F} \vec{q} \int d^2k f_{\vec{k},\vec{k}+\vec{q}}^{c,c} \delta(\epsilon_F - \epsilon_{\vec{k},c}) \delta(\epsilon_F - \epsilon_{\vec{k}+\vec{q},c}). \quad (12)$$

Note that in this limit q is restricted to $q < 2k_F$. To perform the integration in \vec{k} , we choose, without loss of generality, $\vec{q} = (q, 0)$ and write

$$u \equiv \cos \theta = \frac{\vec{k} \cdot \vec{q}}{kq},$$

$$\int d^2k = 2k_F \int_0^\infty dk \int_{-1}^1 \frac{du}{\sqrt{1-u^2}},$$

$$\delta(\epsilon_F - \epsilon_{\vec{k},c}) = \frac{1}{\hbar v_F} \delta(k - k_F),$$

$$\delta(\epsilon_F - \epsilon_{\vec{k}+\vec{q},c}) = \frac{1}{\hbar v_F q} \delta\left(u + \frac{q}{2k_F}\right).$$

Therefore the following result is obtained

$$\vec{\Gamma}(\vec{q}, \omega) = g \frac{\omega \tau_F}{\pi \hbar v_F} \frac{\vec{q}}{q} \left[\frac{f_{\vec{k}, \vec{k}+\vec{q}}^{c,c}}{\sqrt{1-u^2}} \right]_{k=k_F, u=-\frac{q}{2k_F}}. \quad (13)$$

This result is central to this paper. It shows that in the limit of low temperature and high density, the non-linear susceptibility is independent of both the energy dispersion relation and the dependence of the transport time on momentum, depending only in the particular form of the overlap factor. Therefore, this result can be readily applied for the case of a 2DEG, graphene, bilayer graphene and other systems. Although it was already pointed out in [11, 12] that the non-linear susceptibility is independent of the momentum dependence of the transport time, in those works this result was obtained for the particular case of massless Dirac electrons. Here, we show that this is a general result also independent of the energy dispersion relation. Note that this result contradicts [10], where different results for the non-linear susceptibility are obtained for different transport times in the low temperature limit.

Since in the low temperature limit we have $\vec{\Gamma}(\vec{q}, \omega) \propto \omega$, the integration in ω in (3) reads

$$\int_0^\infty \frac{d\omega \omega^2}{\sinh^2(\beta \hbar \omega / 2)} = 2^3 \left(\frac{k_B T}{\hbar} \right)^3 \frac{\pi^2}{6}, \quad (14)$$

which gives the T^2 dependence of the drag conductivity and resistivity in the low temperature limit. The T^2 behaviour is independent of the details of the energy dispersion relation, the transport time and the wave function overlap factors. Notice, however, that the T^2 behaviour might be modified if one includes corrections to the drag conductivity due to higher order terms in the interlayer interaction [20].

3.1. General system at large interlayer distance and strong screening

We now assume that the interlayer separation is large, $k_F d \gg 1$. The interlayer Coulomb interaction decays exponentially with d , thus the integration kernel of (3) is dominated by values of q such that $q \lesssim d^{-1}$. Therefore the condition $k_F d \gg 1$ allow us to expand the remaining integration kernel to lowest order in q . To lowest order, the overlap factor $f_{\vec{k}, \vec{k}+\vec{q}}^{c,c}$ is 1. Therefore at low temperature and for small q and ω , with $\omega < v_F q$, we have

$$\vec{\Gamma}(\vec{q}, \omega) = g \frac{\omega \tau_F}{\pi \hbar v_F} \frac{\vec{q}}{q}, \quad (15)$$

a universal result that is independent of all the details of the system. Note, that although it is clear that in this limit $\vec{\Gamma}$ should only depend on quantities defined at the Fermi level (k_F, τ_F), it is not obvious at first that changing the momentum dependence of $\epsilon_{\vec{k}}$ or $\tau_{\vec{k}}$ will not change the momentum dependence of $\vec{\Gamma}$. For small q we approximate $\chi_a(q, 0) \simeq -\rho_a(\epsilon_{Fa})$, where $\rho_a(\epsilon)$ is the density of states of layer a , and the RPA dielectric function (9) becomes

$$\epsilon_{RPA}(q, 0) = 1 + \frac{q_{TF1} q_{TF2}}{q^2} 2 \sinh(qd) e^{-qd} + \frac{q_{TF1} + q_{TF2}}{q}, \quad (16)$$

with $q_{TFa} = \rho_a(\epsilon_{Fa})e^2/(2\epsilon_0)$, the Thomas-Fermi screening momentum in 2D of layer a . If we assume that we have strong screening, $q_{TF1(2)}d \gg 1$, we further approximate [13]

$$\epsilon_{RPA}(q, 0) = 2 \frac{q_{TF1}q_{TF2}}{q^2} \sinh(qd) e^{-qd}. \quad (17)$$

If the dispersion relation of layer a is given by a power law, $\epsilon_{\vec{k},c}^a = C_a k^{\beta_a}$, then we have $q_{TFa} \propto k_{Fa}^{2-\beta_a}$. Therefore, for a linear dispersion relation the condition $q_{TFa}d \gg 1$ is equivalent to $k_{Fa}d \gg 1$; while for a parabolic dispersion relation q_{TFa} is independent of k_{Fa} , and therefore $q_{TFa}d \gg 1$ becomes an extra assumption. Assuming $q_{TF1(2)}d \gg 1$, and using (15) and (17) in (3) we obtain the following expression for the drag conductivity:

$$\sigma_D = \frac{e_1 e_2}{\hbar} \frac{\zeta(3) g_1 g_2}{2^4} \frac{e_1^2}{4\pi\epsilon_0 v_{F1} \hbar} \frac{e_2^2}{4\pi\epsilon_0 v_{F2} \hbar} \frac{\tau_{F1} \tau_{F2} (k_B T)^2}{\hbar^2 (q_{TF1} d)^2 (q_{TF2} d)^2}. \quad (18)$$

This expression is valid for $\epsilon_{F1(2)}\beta$, $k_{F1(2)}d$, $q_{TF1(2)}d \gg 1$ and is universal in the sense that it does not depend on the particular forms of the energy dispersion relations, transport time dependence on momentum or wave function structure. We obtain the familiar 2DEG T^2 and d^{-4} behaviour for the drag conductivity, proving that it is indeed a much more general result. If the metallic layers are immersed in a homogeneous dielectric medium, with dielectric constant ϵ_r , one should multiply (18) by ϵ_r^2 . Now, we notice that in the low temperature limit the intralayer conductivity for isotropic systems is given by the Boltzmann result

$$\sigma_{aa} = \frac{e_a^2}{2} \rho(\epsilon_{Fa}) v_F^2 \tau_F, \quad (19)$$

where the factor of $1/2$ comes from the fact that we are in two dimensions, the density of states at the Fermi energy is given by

$$\rho(\epsilon_{Fa}) = \frac{g_a}{2\pi} \frac{k_{Fa}}{\hbar v_{Fa}}, \quad (20)$$

and that the carrier density is related to the Fermi momentum in two dimensions by

$$k_{Fa} = \sqrt{\frac{4\pi n_a}{g_a}}. \quad (21)$$

This allow us to express the drag resistivity in terms of the carrier densities as

$$\rho_D = -\frac{\hbar}{e_1 e_2} \frac{\zeta(3)}{2^6 \pi \sqrt{g_1 g_2}} \left(\frac{4\pi\epsilon_0}{e_1^2} \right) \left(\frac{4\pi\epsilon_0}{e_2^2} \right) \frac{(k_B T)^2}{n_1^{3/2} n_2^{3/2} d^4}. \quad (22)$$

It is also usual to express the drag resistivity in this limit in terms of the Fermi energy, momentum and Thomas-Fermi screening momentum. To do this we assume a power law energy dispersion relation, $\epsilon_{\vec{k},c}^a(k) = C_a k^{\beta_a}$, obtaining

$$\rho_D = -\frac{\hbar}{e_1 e_2} \frac{\zeta(3) \pi^2}{\beta_1 g_1 \beta_2 g_2} \frac{(k_B T)^2}{\epsilon_{F1} \epsilon_{F2}} \frac{1}{(k_{F1} d) (k_{F2} d) (q_{TF1} d) (q_{TF2} d)}. \quad (23)$$

For drag between two 2DEG, $\beta_{1(2)} = 2$, $g_{1(2)} = 2$ (spin degeneracy), we re-obtain the known formula from [13]. For graphene we obtain exactly the same result, since

$\beta_{1(2)} = 1$, $g_{1(2)} = 4$ (spin and valley degeneracy). Finally, for the case where each of the two layers are formed by graphene bilayers, $\beta_{1(2)} = 2$, $g_{1(2)} = 4$ (spin and valley degeneracy), we have an extra factor of $\frac{1}{4}$.

3.2. The case of graphene at small interlayer distance

Now we specialize to the case where both metallic layers are formed by single layer graphene, SLG, and analyze the behaviour of the drag conductivity when the layer separation is small, $k_F d \ll 1$. In this situation we can no longer expand the non-linear susceptibilities for small q and need to consider its full dependence on q . In graphene the wave function overlap factor is

$$f_{\vec{k}, \vec{k}+\vec{q}}^{\lambda, \lambda'} = \frac{1}{2} \left(1 + \lambda \lambda' \frac{\vec{k} \cdot (\vec{k} + \vec{q})}{|\vec{k}| |\vec{k} + \vec{q}|} \right), \quad (24)$$

with $\lambda = +, -$ for the conduction and valence band, respectively. Therefore, the non-linear susceptibility in the low temperature limit (13) reads

$$\vec{\Gamma}_{SLG}(\vec{q}, \omega) = 4 \frac{\omega \tau_F}{\pi \hbar v_F} \frac{\vec{q}}{q} \sqrt{1 - \left(\frac{q}{2k_F} \right)^2}, \quad (25)$$

where the factor of 4 comes from the spin and valley degeneracies. Equation (25) is in disagreement with the expressions obtained in [10] both for the momentum independent and for the linearly momentum dependent transport time cases. However, we emphasize that in the low temperature limit (25) holds for an arbitrary transport time. Now we notice that for $q < 2k_F$, the static polarizability for graphene is given by [21]

$$\chi_{SLG}(q < 2k_F, 0) = -\frac{2k_F}{\pi \hbar v_F} = -q_{TF} \frac{2\epsilon_0}{e^2}. \quad (26)$$

Therefore the dielectric function $\epsilon_{RPA}(q, 0)$ still has the form given by (16), even if we do not assume that q is small. Since we have $k_{F1(2)}d \ll 1$, we expand to first order in d

$$\begin{aligned} |U_{12}(q, 0)|^2 &= \left| \frac{V_{12}(q)}{\epsilon_{RPA}(q, 0)} \right|^2 \\ &\simeq \left(\frac{e^2}{2\epsilon_0} \right)^2 \left[\frac{1}{(q + q_{TF1} + q_{TF2})^2} - d \frac{2(2q_{TF1}q_{TF2} + q(q + q_{TF1} + q_{TF2}))}{(q + q_{TF1} + q_{TF2})^3} \right], \end{aligned} \quad (27)$$

and the drag conductivity becomes

$$\sigma_D = \frac{e^2}{\hbar} \frac{2^3}{3} \alpha_g^2 \frac{\tau_F^2 (k_B T)^2}{\hbar^2} [\mathcal{I}^{(0)}(k_{F1}, k_{F2}) - d \mathcal{I}^{(1)}(k_{F1}, k_{F2})], \quad (28)$$

where $\alpha_g = e^2/(4\pi\epsilon_0 v_F \hbar)$ is the fine structure constant of graphene and we have defined the functions

$$\begin{aligned} \mathcal{I}^{(0)}(k_{F1}, k_{F2}) &= \int_0^K \frac{dq q}{(q + (q_{TF1} + q_{TF2}))^2} \sqrt{1 - \frac{q^2}{4k_{F1}^2}} \sqrt{1 - \frac{q^2}{4k_{F2}^2}} \\ \mathcal{I}^{(1)}(k_{F1}, k_{F2}) &= \int_0^K dq \frac{2(2q_{TF1}q_{TF2} + q(q + q_{TF1} + q_{TF2}))}{(q + q_{TF1} + q_{TF2})^3} \sqrt{1 - \frac{q^2}{4k_{F1}^2}} \sqrt{1 - \frac{q^2}{4k_{F2}^2}}, \end{aligned} \quad (29)$$

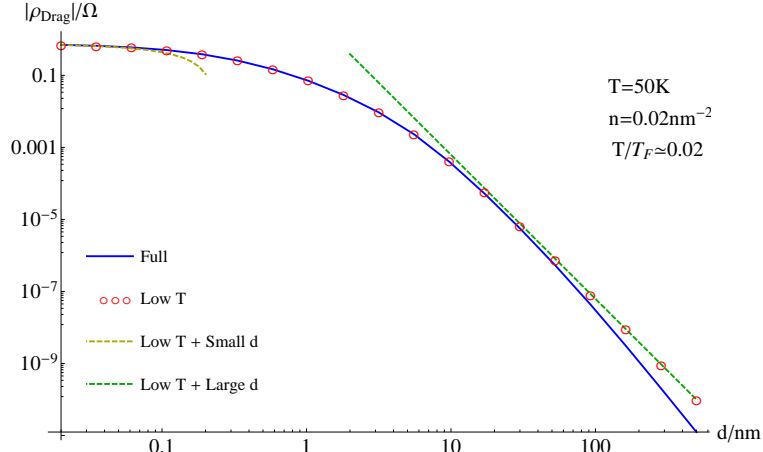


Figure 1. Comparison between computed drag resistivity for graphene in vacuum using the full expression from equations (33) and (34), and several asymptotic limits as a function of the interlayer spacing. The curve *Low T* was computed setting $y = 0$ in ϵ_{RPA} and expanding to first order in y the functions Φ in (34). The curve *Low T + Small d* was computed using (30) and *Low T + Large d* using (22). We can see that the agreement between the full expression and the low temperature one is good for most of the range, but that the small discrepancy is enhanced as d increases.

where $K = 2 \min(k_{F1}, k_{F2})$, and for graphene $q_{TF} = 4\alpha_g k_F$. The drag resistivity becomes

$$\rho_D = -\frac{\hbar}{e^2} \frac{2^3 \pi^2}{3} \frac{(k_B T)^2}{\epsilon_{F1} \epsilon_{F2}} \alpha_g^2 [\mathcal{I}^{(0)}(k_{F1}, k_{F2}) - d \mathcal{I}^{(1)}(k_{F1}, k_{F2})]. \quad (30)$$

Identical result for $d = 0$ has recently been derived in an independent work [12]. For the case where both layers are at the same carrier density $k_{F1} = k_{F2} = k_F$ the functions $\mathcal{I}^{(0)}$ and $\mathcal{I}^{(1)}$ simplify considerably and we obtain,

$$\mathcal{I}^{(0)}(k_F, k_F) = 12\alpha_g - \frac{3}{2} + (1 - 48\alpha_g^2) \log \left(1 + \frac{1}{4\alpha_g} \right) \quad (31)$$

$$\mathcal{I}^{(1)}(k_F, k_F) / k_F = \frac{2}{3} + 44(1 - 8\alpha_g)\alpha_g + \frac{2}{1 + 4\alpha_g} + 32\alpha_g (44\alpha_g^2 - 1) \log \left(1 + \frac{1}{4\alpha_g} \right),$$

Therefore, for drag between two SLG layers as the interlayer separation is increased, the behaviour of the drag resistivity changes from a linear dependence on d for $q_{TF}d \ll 1$ to a d^{-4} dependence at large $q_{TF}d \gg 1$. In figure 1, we can see that the small separation expression (30) is only reliable for $q_{TF}d \lesssim 0.2$ and the large separation expression (22) for $q_{TF}d \gtrsim 20$. If the graphene layers are immersed in a homogeneous dielectric with constant ϵ_r this would correspond to $k_F d \lesssim 0.02\epsilon_r$ and $k_F d \gtrsim 2\epsilon_r$, respectively. We therefore see, that these limits are not easy to be achieved experimentally.

4. Drag in graphene: general formula and asymptotic limit

As we have just seen, the dependence on momentum of the transport time is irrelevant in the low temperature and high density limits, but it will be important in general

however. The dependence of the transport time on momentum in graphene depends on the dominant scattering mechanism. Both for strong short-range impurities (resonant scatterers) and Coulomb impurities the transport time depends linearly on the momentum, $\tau_{\vec{k},\lambda} = \tau^0 |k|$, where τ^0 is a constant with units of length \times time [15]. The linear dependence of the transport time is assumed in [8] and we make the same assumption here. In this case, the non-linear susceptibility for graphene becomes:

$$\begin{aligned} \vec{\Gamma}(\vec{q}, \omega) = & -8\pi v_F \tau^0 \sum_{\lambda, \lambda'} \int \frac{d^2 k}{(2\pi)^2} f_{\vec{k}, \vec{k}+\vec{q}}^{\lambda, \lambda'} \left(n_F(\epsilon_{\vec{k}, \lambda}) - n_F(\epsilon_{\vec{k}+\vec{q}, \lambda'}) \right) \\ & \times \left((\lambda - \lambda') \vec{k} - \lambda' \vec{q} \right) \delta \left(\epsilon_{\vec{k}, \lambda} - \epsilon_{\vec{k}+\vec{q}, \lambda'} + \hbar\omega \right). \end{aligned} \quad (32)$$

We will follow the steps of [8] and assume that both layers are with high electron doping, so that the existence of the valence band can be ignored. In this case, we take only the $\lambda, \lambda' = +, +$ contribution into account. Taking the non-linear susceptibilities at zero temperature, the drag resistivity can be written as

$$\rho_D = -\frac{1}{2^5} \frac{\hbar}{e^2} \frac{\sqrt{\epsilon_{F1}\epsilon_{F2}}}{k_B T} \alpha_g^2 \mathcal{F}(k_{F1}, k_{F2}, d). \quad (33)$$

The function \mathcal{F} is defined as

$$\begin{aligned} \mathcal{F}(k_{F1}, k_{F2}, d) = & \int_0^\infty dx x^3 \int_0^\infty \frac{dy}{\sinh^2 \left(y \frac{\hbar v_F \sqrt{k_{F1} k_{F2}}}{2k_B T} \right)} \frac{e^{-2d\sqrt{k_{F1} k_{F2}} x}}{\epsilon_{12}^2 |\epsilon_{RPA}(x, y)|^2} \\ & \times \frac{\Phi_1(x, y) \Phi_2(x, y)}{1 - \left(\frac{y}{x} \right)^2}, \end{aligned} \quad (34)$$

where the functions $\Phi_a(x, y)$ are introduced in Appendix B and $x = q/\sqrt{k_{F1} k_{F2}}$ and $y = q/(v_F \sqrt{k_{F1} k_{F2}})$. This is exactly the same expression derived in [8] if one notices that the function $\epsilon(x, y)$ used there is related to $\epsilon_{RPA}(x, y)$ by $\epsilon(x, y) = x^2 (x^2 - y^2) \epsilon_{RPA}(x, y)$. So far no approximation has been made in the sense that no asymptotic limit has been considered. A comparison of the different asymptotic behaviours computed in the previous section with the exact result, (33), is given in figure 1.

By the general arguments given in the previous section, the drag resistivity in the low temperature, high density, large separation limit should behave as d^{-4} . However, in [8] and in this same limit a dependence of d^{-6} was obtained. In [10], the d^{-6} result is attributed to scaling of the vertex function $\vec{v}_{\vec{k}, \lambda} \tau_{\vec{k}, \tau}$ used in [8] with q^2 , while it should scale with q for a constant group velocity and a transport time linearly dependent on the momentum, implying that the carrier group velocity used in [8] depends linearly on momentum. We clarify that in [8], as in this paper, the group velocity used is constant and the transport time depends linearly in momentum, so that the vertex function depends linearly in momentum, $\vec{v}_{\vec{k}, \lambda} \tau_{\vec{k}, \lambda} = \lambda v_F \tau^0 \vec{k}$. However, as we have argued in the previous section, the momentum dependence of the vertex function is irrelevant in this limit. The incorrect d^{-6} dependence was obtained in [8] due to an error at the end of the asymptotic calculation: for small x and y with $y < x$ the function $\Phi(x, y)$ was taken to behave as $\Phi(x, y) \sim \frac{y}{x}$, when it actually behaves as $\Phi(x, y) \sim \frac{y}{x^2}$. This

changes the integration kernel obtained in [8] in the asymptotic limit from $q^5 \sinh^{-2}(qd)$ to $q^3 \sinh^{-2}(qd)$, which changes the dependence from d^{-6} to the correct behaviour of d^{-4} . Therefore, for $\epsilon_{F1(2)}\beta$, $k_{F1(2)}d \gg 1$, the exact (33) and (34) give

$$\rho_D = -\frac{\hbar}{e^2} \frac{\zeta(3)\pi^2 (k_B T)^2}{2^4} \frac{1}{\epsilon_{F1}\epsilon_{F2} (k_{F1}d) (k_{F2}d) (q_{TF1}d) (q_{TF2}d)}, \quad (35)$$

in agreement with the general result discussed in the previous section. Expressing (35) in terms of the carrier density we get

$$\rho_D = -\frac{\hbar}{e^2} \frac{\zeta(3)}{2^8\pi} \frac{1}{\alpha_g^2} \frac{(k_B T)^2}{(v_F \hbar)^2 n_1^{3/2} n_2^{3/2} d^4}, \quad (36)$$

as obtained before, but here we have started from the general expression for the drag, that is, (33). Note that, in this limit the drag resistivity decreases as α_g is increased. We can understand this as follows: as α_g increases the screening becomes more effective making the momentum transfer between layers less effective.

5. Comparison with experiments

In the experimental setup of [3] we have two graphene layers, which we denote by t (top) and b (bottom). Between the two graphene sheets we have a layer of Al_2O_3 , thickness $d = d_t = 7$ nm (in [8] it was used a value of 14 nm). The bottom layer is on top of a $d_b = 280$ nm thickness SiO_2 . Finally, these layers are on top of a silicon wafer. The carrier density in the graphene sheets is controlled using a back gate voltage between the silicon wafer and the bottom layer. For the relation between the gate voltage and the carrier densities the reader is referred to [3, 8]. Given the carrier densities, one can compute the drag resistivity using (33) and (34). For the fine structure constant of graphene we will use the accepted value of $\alpha_g = 2$. To compute the drag, we must determine the form of the bare Coulomb interactions, $V_{ab}(\vec{q})$ (7), with $a, b = t, b$. In a naive treatment of the Coulomb interactions one could write [8]:

$$\epsilon_{tt}^{(naive)} = \frac{\epsilon_{air} + \epsilon_{Al_2O_3}}{2} \quad \epsilon_{tb}^{(naive)} = \epsilon_{Al_2O_3} \quad \epsilon_{bb}^{(naive)} = \frac{\epsilon_{Al_2O_3} + \epsilon_{SiO_2}}{2} \quad (37)$$

with ϵ_{air} , $\epsilon_{Al_2O_3}$ and ϵ_{SiO_2} the dielectric constants of air, Al_2O_3 and SiO_2 , respectively. Using the values of $\epsilon_{air} = 1$, $\epsilon_{SiO_2} = 3.8$ and $\epsilon_{Al_2O_3} = 5.6$ [22] we computed the drag resistivity curves with this model for the interactions. In figure 2(a), we can see the comparison between the experimental results from [3] and this calculation. Clearly the agreement is not very good. Although (37) is correct in some limits, to obtain the exact form of the Coulomb interactions in layered dielectric medium one must solve Poisson's equation (see Appendix A). For a 3 layered dielectric structure, for the effective dielectric constants $\epsilon_{ab}(q)$ in (7) we would have to use the functions $\epsilon_{ab}^{(3)}(q)$ defined in Appendix A, with $\epsilon_1 = \epsilon_{air} = 1$, $\epsilon_2 = \epsilon_{Al_2O_3} = 5.6$ and $\epsilon_3 = \epsilon_{SiO_2} = 3.8$. In figure 2(b), we can see the comparison between the computed resistivity using this interactions and the experimental results. Although this is a more rigorous treatment for the interactions than the naive one, (37), the computed drag curves deviate even

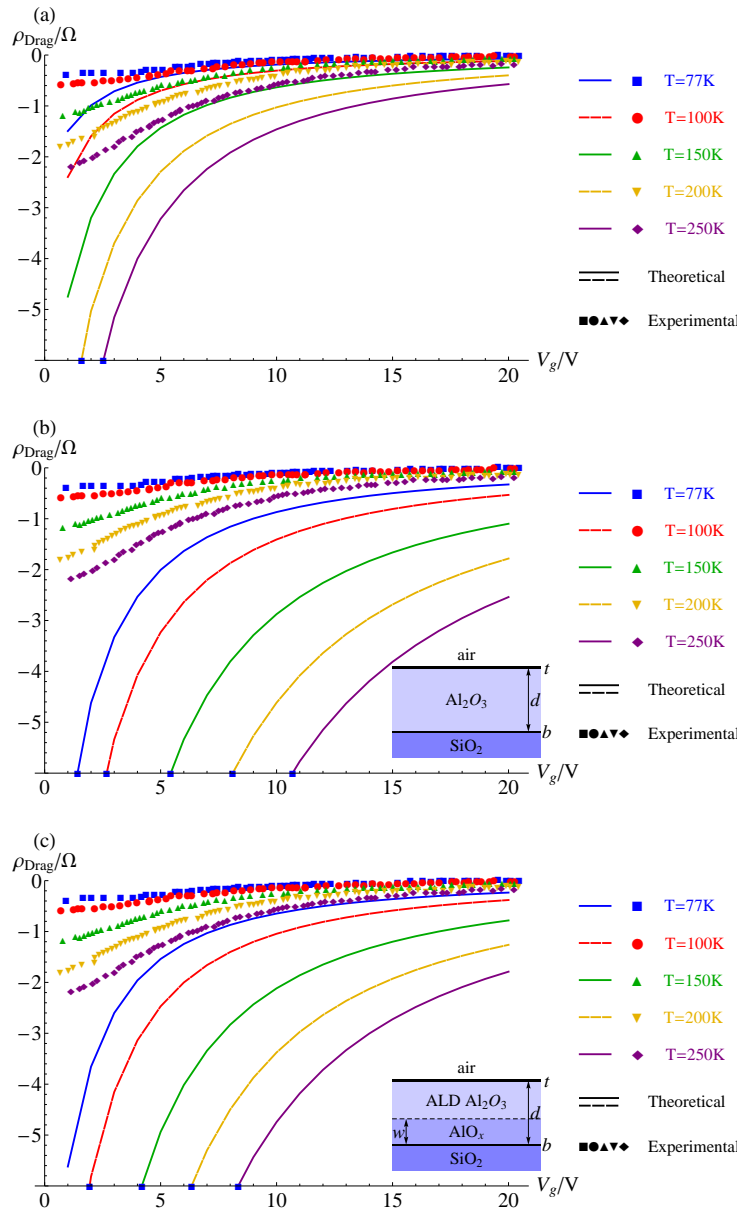


Figure 2. Comparison between computed and measured drag resistivity. The dotted marks are the experimental results from [3]. The solid lines are the computed curves. Curves with the same colour correspond to the same temperature. (a) The computed curves were calculated using the naive interactions $(\epsilon_{ab}^{(naive)})$. (b) The computed curves were calculated using the Coulomb interactions obtained by solving Poisson's equation in a 3 layered dielectric depicted in the inset ($\epsilon_{ab}^{(3)}(q)$). (c) The computed curves were calculated using the Coulomb interactions obtained by solving Poisson's equation in a 4 layered dielectric as depicted in the inset ($\epsilon_{ab}^{(4)}(q)$). The value used for the fine structure of graphene, α_g , was $\alpha_g = 2$ in all plots. No attempt to fit the data was made.

further from the experimental results. One could proceed as in [8] and use α_g as a fitting parameter. We found, however, that it would be necessary a value of the order $\alpha_g \sim 5$ and $\alpha_g \sim 16$, for the interactions with $\epsilon_{ab}^{(naive)}$ and $\epsilon_{ab}^{(3)}$, respectively, to fit the data; both values are unrealistic. Taking into account that the SiO_2 layer is finite when solving Poisson's equation has virtually no effect in the computed drag resistivity. This situation is puzzling at least. However, more attention must be paid to how the devices from [3] are constructed. The Al_2O_3 dielectric layer is deposited in two steps. (i) First a 2 nm aluminium layer is deposited on top of the bottom graphene layer, being later oxidized. We will refer to this as AlO_x layer. (ii) On top of the oxidized aluminium layer, a 5 nm Al_2O_3 layer is directly deposited through atomic layer deposition. We will refer to this as ALD layer. Completed the procedure a 7 nm Al_2O_3 is obtained. The reason the Al_2O_3 is not directly deposited on top of graphene is due to graphene being chemically inert. In [22], the dielectric constant of the ALD alumina was determined by studying how the capacitance of devices similar to the ones from [3] scales with the tickness of the ALD layer keeping the AlO_x layer thickness fixed, obtaining the already referred value of $\epsilon_{\text{Al}_2\text{O}_3} = 5.6$. The AlO_x layer contributed with a finite capacitance, which corresponds to a dielectric constant of $\epsilon_{\text{AlO}_x} = 2.7$. The reason why ϵ_{AlO_x} differs from $\epsilon_{\text{Al}_2\text{O}_3}$ is not clear but it is most likely due to interface effects between the graphene layer and the dielectric or between the ALD and the AlO_x layers. In this case, it is not clear if we can attribute a bulk value for ϵ_{AlO_x} . Nevertheless, we still considered this situation. For this situation of a 4 layered dielectric we use the functions $\epsilon_{ab}^{(4)}(q)$ from Appendix A, now with $\epsilon'_2 = \epsilon_{\text{AlO}_x} = 2.7$ and $w = 2$ nm. The comparison between the drag curves computed this way and the experimental data can be seen in figure 2(c). We can see that there is a better approximation with the experimental results, but this should be regarded with caution.

6. Conclusions

In this paper we have showed that in the limit of low temperature, high density, large interlayer distance and strong screening, the drag resistivity should always behave with T^2 and d^{-4} . This result was obtained for general dispersion relation, momentum dependence of the transport time and electronic wave function structure. It is therefore a more general result than previous ones. Central to this fact is the general expression derived for the non-linear susceptibility, (13), in the low temperature limit to lowest order in frequency. Being general, this result also applies to graphene, and should close the ongoing debate regarding the behaviour of drag for this system in the aforementioned limit. We also derived an asymptotic expression for drag in graphene in the limit of low temperature, high density and small layer separation. Finally, we compared the available experimental data on drag in graphene from [3] with our theoretical model and found out that if one insists in the conventional value for graphene fine structure constant, $\alpha_g = 2$, a quantitative understanding of the data is still lacking. This is most likely due to the complexity of the dielectric substrate used and one hopes that the

theoretical model will be quantitatively more successful in devices built using simpler substrates, such as boron nitride [23]. Another possibility to account for this discrepancy could be spacial inhomogeneities in the chemical potential of the graphene layers, which are not taken into account with the present formalism. Although, it is not clear if these should be important in the experimental range investigated, spacial inhomogeneities in graphene's carrier density will for sure play an important role in the electron-hole puddle regime and should be object of future work.

Acknowledgments

We thank useful discussions with F. Guinea, J.M.B. Lopes dos Santos and A.H. Castro Neto. Special thanks to E. Tutuc for helpful discussions regarding the construction of graphene double layer structures. B. Amorim was supported by Fundação para a Ciência e a Tecnologia (FCT) through Grant No. SFRH/BD/78987/2011.

Appendix A. Electron-electron Coulomb interaction

To obtain the correct form of the electron-electron interaction in a dielectric material one must solve Poisson's equation, $-\nabla(\epsilon\nabla\phi) = \rho_{free}/\epsilon_0$, where ρ_{free} is the free charge density. We are interested in situations where the dielectric constant is a piecewise constant function of the z coordinate, with discontinuities at positions z_i . Since, we want the interaction matrix element between states with well defined momentum in the x, y directions and well defined position in the z direction, it is useful to introduce $\phi(q, z) = \int d^2x \phi(\vec{x}, z) e^{-i\vec{q}\cdot\vec{x}}$, where $\vec{q} = (q_x, q_y)$ and $\vec{x} = (x, y)$. For a point charge located at $(\vec{x}, z) = (\vec{0}, z_p)$, $\rho_{free} = e\delta(\vec{0})\delta(z_p)$, we obtain the following equation

$$-\frac{\partial}{\partial z} \left(\epsilon(z) \frac{\partial}{\partial z} \phi(q, z) \right) + q^2 \phi(q, z) = \frac{e}{\epsilon_0} \delta(z_p). \quad (\text{A.1})$$

The potential $\phi(q, z)$ is continuous everywhere, and the the function $\epsilon(z) \frac{\partial \phi(q, z)}{\partial z}$ is continuous except at the position of the point charge. Therefore we have the boundary conditions

$$\begin{aligned} \phi(q, z_i^+) &= \phi(q, z_i^-), \\ \epsilon(z_i^-) \frac{\partial \phi(q, z_i^-)}{\partial z} - \epsilon(z_i^+) \frac{\partial \phi(q, z_i^+)}{\partial z} &= \frac{e}{\epsilon_0} \delta_{z_i, z_p}. \end{aligned} \quad (\text{A.2})$$

Solving equation (A.1) together with the boundary conditions (A.2) and imposing that $\phi(q, z)$ decays at infinity, we can obtain the potential created by the point charge and from that the bare electron-electron Coulomb interaction. Let us consider that the metallic plates are located at $z = 0$ and $z = d$. The electron-electron interaction can be cast in the form given in (7) with $a, b = t, b$, where t refers to the top layer and b refers

to the bottom layer. For a 3 layered dielectric:

$$\epsilon^{(3)}(z) = \begin{cases} \epsilon_1 & , z > d \\ \epsilon_2 & , d > z > 0, \\ \epsilon_3 & , 0 > z \end{cases} \quad (\text{A.3})$$

solving Poisson's equation gives

$$\begin{aligned} \epsilon_{tt}^{(3)}(q) &= \frac{e^{2qd}(\epsilon_3 + \epsilon_2)(\epsilon_1 + \epsilon_2) - (\epsilon_3 - \epsilon_2)(\epsilon_1 - \epsilon_2)}{2(1 + e^{2qd})\epsilon_2 - 2(1 - e^{2qd})\epsilon_3}, \\ \epsilon_{tb}^{(3)}(q) &= \frac{e^{2qd}(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3) - (\epsilon_1 - \epsilon_2)(\epsilon_3 - \epsilon_2)}{4e^{2qd}\epsilon_2}, \\ \epsilon_{bb}^{(3)}(q) &= \frac{e^{2qd}(\epsilon_1 + \epsilon_2)(\epsilon_3 + \epsilon_2) - (\epsilon_1 - \epsilon_2)(\epsilon_3 - \epsilon_2)}{2(1 + e^{2qd})\epsilon_2 - 2(1 - e^{2qd})\epsilon_1}. \end{aligned} \quad (\text{A.4})$$

This result was previously given in [9] and [24]. If we consider a 4 layered dielectric,

$$\epsilon^{(4)}(z) = \begin{cases} \epsilon_1 & , z > d \\ \epsilon_2 & , d > z > w \\ \epsilon'_2 & , w > z > 0 \\ \epsilon_3 & , 0 > z \end{cases} \quad (\text{A.5})$$

we obtain,

$$\begin{aligned} \epsilon_{tt}^{(4)}(q) &= \frac{D^{(4)}(q)}{2} \left[e^{2q(d+w)} (\epsilon_2 + \epsilon'_2) (\epsilon_3 + \epsilon'_2) + e^{2dq} (\epsilon_2 - \epsilon'_2) (\epsilon'_2 - \epsilon_3) \right. \\ &\quad \left. + e^{4qw} (\epsilon_2 - \epsilon'_2) (\epsilon_3 + \epsilon'_2) + e^{2qw} (\epsilon_2 + \epsilon'_2) (\epsilon'_2 - \epsilon_3) \right]^{-1}, \\ \epsilon_{tb}^{(4)}(q) &= \frac{D^{(4)}(q)}{8e^{2q(d+w)}\epsilon_2\epsilon'_2}, \\ \epsilon_{bb}^{(4)}(q) &= \frac{D^{(4)}(q)}{2} \left[e^{2q(d+w)} (\epsilon_1 + \epsilon_2) (\epsilon_2 + \epsilon'_2) - e^{2dq} (\epsilon_1 + \epsilon_2) (\epsilon_2 - \epsilon'_2) \right. \\ &\quad \left. + e^{4qw} (\epsilon_2 - \epsilon'_2) (\epsilon_3 + \epsilon'_2) - e^{2dq} (\epsilon_1 + \epsilon_2) (\epsilon_2 - \epsilon'_2) \right]^{-1}, \end{aligned} \quad (\text{A.6})$$

where $D^{(4)}(q)$ is defined as

$$\begin{aligned} D^{(4)}(q) &= e^{2q(d+w)} (\epsilon_1 + \epsilon_2) (\epsilon_2 + \epsilon'_2) (\epsilon'_2 + \epsilon_3) + e^{2dq} (\epsilon_1 + \epsilon_2) (\epsilon_2 - \epsilon'_2) (\epsilon'_2 - \epsilon_3) \\ &\quad + e^{4qw} (\epsilon_1 - \epsilon_2) (\epsilon_2 - \epsilon'_2) (\epsilon'_2 + \epsilon_3) + e^{2qw} (\epsilon_1 - \epsilon_2) (\epsilon_2 + \epsilon'_2) (\epsilon'_2 - \epsilon_3). \end{aligned} \quad (\text{A.7})$$

Appendix B. Details of the computation of the drag resistivity in graphene

The contribution to the non-linear susceptibility of graphene (32) coming only from the conductance band, $\lambda, \lambda' = +, +$ is

$$\begin{aligned} \vec{\Gamma}(\vec{q}, \omega) &= 8\pi v_F \tau^0 \vec{q} \int \frac{d^2k}{(2\pi)^2} f_{\vec{k}, \vec{k}+\vec{q}}^{+,+} \left(n_F(\epsilon_{\vec{k},+}) - n_F(\epsilon_{\vec{k}+\vec{q},+}) \right) \delta \left(\epsilon_{\vec{k},+} - \epsilon_{\vec{k}+\vec{q},+} + \hbar\omega \right) \\ &= -2v_F \tau^0 \vec{q} \text{Im} \chi^{++}(q, \omega) \end{aligned} \quad (\text{B.1})$$

where $\chi^{++}(q, \omega)$ is the contribution to the graphene polarizability coming only from the conductance band,

$$\chi^{++}(q, \omega) = 4 \int \frac{d^2k}{(2\pi)^2} f_{\vec{k}, \vec{k}+\vec{q}}^{+,+} \frac{n_F(\epsilon_{\vec{k},+}) - n_F(\epsilon_{\vec{k}+\vec{q},+})}{\epsilon_{\vec{k},+} - \epsilon_{\vec{k}+\vec{q},+} + \hbar\omega + i0^+}. \quad (\text{B.2})$$

At zero temperature, this can be computed analytically and the result can be written as

$$\text{Im}\chi^{++}(\vec{q}, \omega) = \frac{1}{4\pi v_F \hbar} \frac{q}{\sqrt{1 - \left(\frac{\omega}{qv_F}\right)^2}} \Phi(q, \omega), \quad (\text{B.3})$$

where the function $\Phi(q, \omega)$ is defined as

$$\begin{aligned} \Phi(q, \omega > 0) &= \Phi^+(q, \omega) \Theta\left(\frac{\omega}{v_F} - q + 2k_F\right) \Theta\left(q - \frac{\omega}{v_F}\right) \\ &+ \Phi^-(q, \omega) \Theta\left(k_F - \frac{\omega}{v_F} - |k_F - q|\right), \end{aligned} \quad (\text{B.4})$$

where

$$\Phi^\pm(q, \omega) = \pm \cosh^{-1} \left(\frac{2k_F \pm \omega/v_F}{q} \right) \mp \frac{2k_F \pm \omega/v_F}{q} \sqrt{\left(\frac{2k_F \pm \omega/v_F}{q} \right)^2 - 1}. \quad (\text{B.5})$$

The drag conductivity therefore becomes

$$\sigma_D = \frac{e^2(\tau^0)^2}{2^7 \pi^4 \hbar^3 k_B T} \int_0^\infty dq q^5 \int_0^\infty \frac{d\omega}{\sinh^2(\beta \hbar \omega / 2)} \frac{|U_{12}(\vec{q}, \omega)|^2}{1 - \left(\frac{\omega}{qv_F}\right)^2} \Phi_1(q, \omega) \Phi_2(q, \omega). \quad (\text{B.6})$$

The intralayer graphene conductivity of layer a at low temperature is given by $\sigma_{aa} = e^2 v_F n_a \tau^0 / \hbar = e^2 \epsilon_F \tau_F / (\pi \hbar^2)$ where $k_{Fa} \tau^0 = \tau_{Fa}$ is the transport time at the Fermi level. We write the interlayer interaction as $U_{12} = e^2 \exp(-qd) / (2\epsilon_{RPA}(\vec{q}, \omega) \epsilon_{12}(q) \epsilon_0 q)$. Introducing the adimensional quantities, $x = q / \sqrt{k_{F1} k_{F2}}$ and $y = \omega / (v_F \sqrt{k_{F1} k_{F2}})$, the drag resistivity becomes

$$\begin{aligned} \rho_D &= -\frac{1}{2^5} \frac{\hbar}{e^2} \frac{\sqrt{\epsilon_{F1} \epsilon_{F2}}}{k_B T} \alpha_g^2 \int_0^\infty dx x^3 \int_0^\infty \frac{dy}{\sinh^2\left(y \frac{\hbar v_F \sqrt{k_{F1} k_{F2}}}{2k_B T}\right)} \\ &\times \frac{e^{-2d\sqrt{k_{F1} k_{F2}} x}}{\epsilon_{12}(q)^2 |\epsilon_{RPA}(x, y)|^2} \frac{\Phi_1(x, y) \Phi_2(x, y)}{1 - \left(\frac{y}{x}\right)^2}, \end{aligned} \quad (\text{B.7})$$

where α_g is the fine structure constant of graphene.

References

- [1] Pogrebinskii M 1977 *Fiz. Tekh. Poluprovodn.* **11** 637 [1977 *Sov. Phys. Semicond.* **11** 372]
- [2] Price P 1983 *Physica (Amsterdam)* B **117** 750
- [3] Kim S, Jo I, Nah J, Yao Z, Banerjee S and Tutuc E 2011 *Phys. Rev. B* **83** 161401
- [4] Geim A 2012 Electronic properties of graphene-bn heterostructures. talk at KITP Conference Fundamental Aspects of Graphene and Other Carbon Allotropes
- [5] Kim S and Tutuc E 2012 *Solid State Communications* **152** 1283 – 1288

- [6] Narozhny B 2007 *Phys. Rev. B* **76** 153409
- [7] Wang-Kong T, Hu B and Das Sarma S 2007 *Phys. Rev. B* **76** 081401
- [8] Peres N, Lopes dos Santos N and Castro Neto A 2011 *EPL (Europhysics Letters)* **95** 18001
- [9] Katsnelson M 2011 *Phys. Rev. B* **84** 041407
- [10] Hwang E, Sensarma R and Das Sarma S 2011 *Phys. Rev. B* **84** 245441
- [11] Narozhny B, Titov M, Gornyi I and Ostrovsky P 2012 *Phys. Rev. B* **85** 195421
- [12] Carrega M, Tudorovskiy T, Principi A, Katsnelson M and Polini M 2012 *ArXiv e-prints (Preprint 1203.6777)*
- [13] Jauho A and Smith H 1993 *Phys. Rev. B* **47** 4420–8
- [14] Gramila T, Eisenstein J, MacDonald A, Pfeiffer L and West K 1991 *Phys. Rev. Lett.* **66** 1216–9
- [15] Peres N 2010 *Rev. Mod. Phys.* **82** 2673–700
- [16] Flensberg K and Hu B 1995 *Phys. Rev. B* **52** 14796–808
- [17] Zheng L and MacDonald A 1993 *Phys. Rev. B* **48** 8203–9
- [18] Kamenev A and Oreg Y 1995 *Phys. Rev. B* **52** 7516–27
- [19] Hu B and Das Sarma S 1993 *Phys. Rev. B* **48** 5469–504
- [20] Levchenko A and Kamenev A 2008 *Phys. Rev. Lett.* **100** 026805
- [21] Wunsch B, Stauber T, Sols F and Guinea F 2006 *New Journal of Physics* **8** 318
- [22] Fallahazad B, Lee K, Lian G, Kim S, Corbet C, Ferrer D, Colombo L and Tutuc E 2012 *Appl. Phys. Lett.* **100** 093112
- [23] Ponomarenko L A *et al* 2011 *Nat. Phys.* **7** 958–61
- [24] Profumo R, Polini M, Asgari R, Fazio R and MacDonald A 2010 *Phys. Rev. B* **82** 085443